1. References

1.1 "Standard Practices for Cycle Counting in Fatigue Analysis", ASTM E 1049-85 (Re-approved 2011).

2. Discussion

2.1 Rainflow

Events are disturbances that change the state of stress or strain. A single event is associated with one or more perturbation, and a single perturbation changes the stress or strain state from t1 (start time) to t2 (end time). This change of stress state is referred to as a half-cycle. Multiple half-cycle taken together give loads spectrum or loads cycles. Damage from a single load cycle is related to the amplitude of the cycle. The amplitude of the cycles is again a function of the sequence of the perturbations. In an environment where the sequence of the events (and thereby perturbations) is not fixed, to estimate the maximum damage it is necessary to re-sequence the events in a manner that the amplitude of the cycles are maximized. Note that only perturbations/half-cycles are re-sequenced (not the order of t1 and t2 in a half-cycle). There are several algorithms for re-sequencing the events/perturbations. Rain-flow algorithm is the most widely used. Though there are some variations of rain-flow algorithm, the one of Reference 1 is used in the BCA EXCEL based fatigue analysis.

Even the implementation of rain-flow algorithm can have variations. In some implementations, a cycle (closed hysteresis loop) is one in which the range of a stress reversal is less than to one before and one after it. In the algorithm used in EXCEL based MACRO, a cycle (closed hysteresis loop) is one in which the range of a stress reversal is less than or equal to one before and one after it. These two approaches can give slightly different results.

As in most procedures, implementation is in terms of full cycles (not half cycles). You can look at it this way. Every event is associated with two time points: t1 and t2. So every event is a half cycle. Say t1 is associated with (mean - osc) and t2 is associated with (mean + osc). When you have a second event, it too has a pair of time points (t1, t2). When you put the two events one after other, you get three (not two) half cycles: t1-t2, (t2-t1), t1-t2. The t2-t1 is actually an implied half-cycle because it is a continuous process. So when you have "n" events you will have "n-1" full cycles and one half-cycle. In rain-flow we assume that the system comes back to its original state. In other words, we add to the end of the vector, the stress associated with t1 of the first event. This addition is actually an addition of a half-cycle. Now we have two half-cycles equal to a full cycle. So in rain-flow you will have only full cycles. There are some algorithms that are in terms of half-cycles and those that do not add an extra point, and the results from those algorithms may differ slightly from those based on full cycles. Salient features of this implementation are:

(a). t1 is associated with (mean - osc) and t2 is associated with (mean + osc)
(b). The vector which is input to the rain-flow algorithm has the highest stress/strain as the first value. The value added to the end of the vector is equal to the first value.
(c). There are no half cycles.
(d). Small amplitude cycles can be eliminated from pre-rain flow vector.
(e). Cycles with frequency larger than two are set aside from rain flow vector.
(f). Non-oscillatory points are eliminated from pre-rain flow vector.

In (a) setting t1 value as (mean + osc) and t2 value as (mean - osc) would make no difference. In (b) setting the first value of input vector as smallest and adding this value at the end of the vector should make no difference. Now it is clear why different implementations of rain-flow may give slightly different results – because some of the features implementations (such as adding an extra point, the value of point added, use of relational operators) are not standard.

2.2 Sample Verification Case 1

Problem Description: The pre-rain-flow stress spectrum is given in Table 1. Re-sequence the stress cycles using rain-flow algorithm.

Table 1: Pre-rainflow stress spectrum

Event	σ_1	σ_2	Cycles
5	+456.96 MPa	-35.56 MPa	1
6	+531.77 MPa	+276.16 MPa	88
7	+415.50 MPa		1

Pre-rainflow stress vector (182 points):

$$\begin{Bmatrix} -35.56 \\ +456.96 \\ -35.56 \\ +415.50 \\ +531.77 \\ +276.16 \\ .... \\ .... \\ .... \\ +531.77 \\ +276.44 \\ +415.50 \\ +415.50 \end{Bmatrix} \text{MPa}$$

Eliminated repeat cycles (86 cycles = 172 points):

Table 2: Eliminated repeated cycles

Event	σ_1	σ_2	Cycles
6	+531.77 MPa	+276.16 MPa	86

Pre-rain flow stress vector after removal of estimated repeated cycles (10 points):

$$\begin{Bmatrix} -35.56 \\ +456.96 \\ -35.56 \\ +415.50 \\ +531.77 \\ +276.16 \\ +531.77 \\ +276.16 \\ +415.50 \\ +415.50 \end{Bmatrix} \text{MPa}$$

In the code, events with zero frequency are always associated with a steady stress (which in this case is 0.0 MPa for events other than 5, 6 and 7). Therefore, the actual pre-rainflow stress vector will have several 0.0 MPa stress points at the beginning (associated with events 1 through 4) and several 0.0 MPa stress points at the end (associated with event 8 through-end).

$$\begin{Bmatrix} 0.0 \\ \ldots \\ 0.0 \\ -35.56 \\ +456.96 \\ -35.56 \\ +415.50 \\ +531.77 \\ +276.16 \\ +531.77 \\ +276.16 \\ +415.50 \\ +415.50 \\ 0.0 \\ \ldots \\ 0.0 \end{Bmatrix} \text{MPa}$$

Re-arranged pre-rain flow stress vector with max value at start and end:

$$\begin{Bmatrix} +531.77 \\ +276.16 \\ +531.77 \\ +276.16 \\ +415.50 \\ +415.50 \\ 0.0 \\ 0.0 \\ \ldots \\ \ldots \\ \ldots \\ -35.56 \\ +456.96 \\ -35.56 \\ +415.50 \\ +531.77 \end{Bmatrix} \text{MPa}$$

Since cycles are associated with stress reversal, eliminate any stress point between a max stress point and a min stress point. Also, remove cycles (not stress point) with amplitude 0.0 MPa. The resultant pre-rainflow stress vector (9 points) is as follows:

$$\begin{Bmatrix} +531.77 \\ +276.16 \\ +531.77 \\ +276.16 \\ +415.50 \\ -35.56 \\ +456.96 \\ -35.56 \\ +531.77 \end{Bmatrix} \text{MPa}$$

The post-rainflow stress spectrum is then computed.

Table 3 shows the extraction cycles by rain flow algorithm from the 9-point pre-rain flow stress spectrum. Figure 1, Figure 2 and Figure 3 show the stress points before the start of any pass for extraction of cycles. Post rain-flow stress spectrum is shown in Table 4.

Table 3: Rainflow stress cycle extraction based on 9 points

Point	Value	Pass 1		Pass 2		Pass 3	
		Status	**Post-spec σ**	**Status**	**Post-spec σ**	**Status**	**Post-spec σ**
a	531.77					remove	4-smax (full)
b	276.16			remove	3-smin (full)		
c	531.77			remove	3-smax (full)		
d	276.16	remove	1-smin (full)				
e	415.50	remove	1-smax (full)				
f	-35.56					remove	4-smax (full)
g	456.96	remove	2-smin (full)				
h	-35.56	remove	2-smax (full)				
a	531.77					remove	4-smax (full)

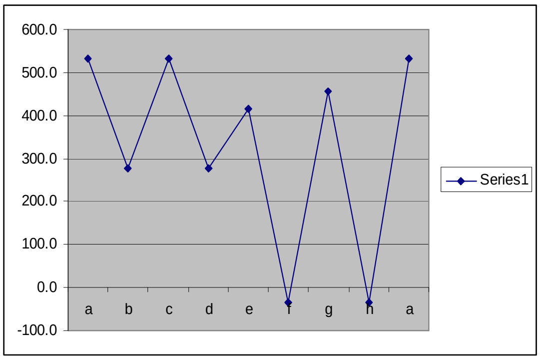

Figure 1: Pass 1 cycle extraction

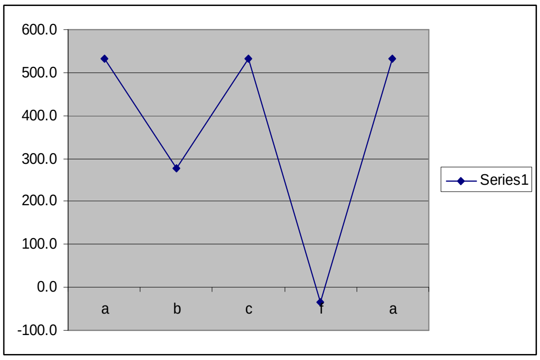

Figure 2: Pass 2 cycle extraction

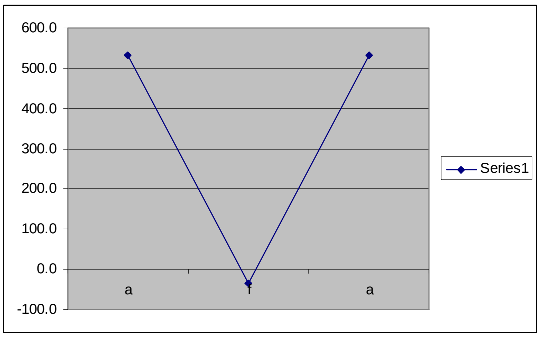

Figure 3: Pass 3 cycle extraction

Table 4: Post rain flow stress spectrum

From retained points			From eliminated points			Total		
σ_1(MPa)	σ_2(MPa)	cycles	σ_1(MPa)	σ_2(MPa)	cycles	σ_1(MPa)	σ_2(MPa)	cycles
415.50	276.16	1				415.50	276.16	1
456.96	-35.56	1				456.96	-35.56	1
531.77	276.16	1				531.77	276.16	1
531.77	-35.56	1				531.77	-35.56	1
			531.77	276.16	86	531.77	276.16	86

Table 5: Re-sequenced vector using rainflow MACRO in EXCEL computations

t1(in)	t2(in)	t1(out)	t2(out)
5.32E+02	2.76E+02	531.77	276.16
5.32E+02	2.76E+02	415.50	276.16
4.16E+02	-3.56E+01	456.96	-35.56
4.57E+02	-3.56E+01	531.77	-35.56

Rainflow correctly re-sequenced the retained points.

2.3 Sample Verification Case 2

Problem description: The pre-rain-flow stress spectrum is given in Table **6**. Re-sequence the stress cycles using rain-flow algorithm.

Table 6: Pre-rainflow stress vector

Pre-rainflow		
Smax	**Smin**	**Cycles**
Point A	-2	1
Point B	1	1
Point C	-3	1
point D	5	1
Point E	-1	1
Point F	3	1
Point G	-4	1
Point H	4	1
Point I	-2	1

The results are presented in Table 7 :

Table 7: Post-rainflow stress vector

Post-rainflow		
Smax	**Smin**	**Cycles**
3	-1	1
1	-2	1
4	-3	1

5	-4	1

Table 8: Re-sequenced vector using rainflow MACRO in HAUS computation

t1(in)	t2(in)	t1(out)	t2(out)
-2.00	1.00	3.00	-1.00
-3.00	5.00	1.00	-2.00
-1.00	3.00	4.00	-3.00
-4.00	4.00	5.00	-4.00

The results are presented in Table 8:

In reference a cycle (closed hysteresis loop) is one in which the range of a stress reversal is less than to one before and one after it. In the algorithm used in EXCEL based MACRO, a cycle (closed hysteresis loop) is one in which the range of a stress reversal is less than or equal to one before and one after it.

2.4 Conclusion

Rainflow algorithm is meant to re-sequence the events and extract the cycles based on the re-sequenced events. Rain flow algorithm as implemented in the EXCEL based procedure does it correctly based on the two verification cases.

3. Actions

Customer to review and approve the rainflow method presented.

4. Attachments

No attachments.